

Distinct Concepts of Zero

A Sept. 1997 Aztlan contribution by John Justeson is included later on this page. Thanks to Randa Marhenke. A 1997 essay from the Atlantic Monthly by David Tisler on Zero through history is accessible here: <http://www.theatlantic.com/issues/97jul/zero.htm>

When one studies quite different mathematical systems in quite different cultures, it becomes evident that "zero" is not as simple a concept as we might at first suppose.

There is a placeholder in a positional notation system. The Maya clearly had that.

There is of course the absence of anything (thus a count of zero).

There is an abstract number "0", between "-1" and "+1".

And there is sometimes ambiguity in cyclic systems between zero, beginning, and completion.

Among Maya glyphs we have at least these:

Where a month is named in the Haab portion of a Calendar Round, there is a distinction between "seating" (beginning, as "0 Mol") and a glyph which appears to combine 'edge' and TUUN (perhaps referring to the last remaining day or bit of a month, as 20 Mol). Mathematically these represent the same day, but with adjacent month names! So they have different meanings. Here are the three examples for which Thompson considered the preservation good enough and the reading secure (his discussion 1960 pp.120-121):

20 Mol = 0 Ch'en (Palenque Temple of the Cross at D9)

20 Yaxk'in = 0 Mol (Piedras Negras, shell plaque, 9.14.17.14.17)

Uayeb completed = 0 Pop (Naranjo Hieroglyphic stairway at A1b)

So there was apparently a day when one month was breathing its last and the next month was being seated? This would be the day which did not carry any of the numbers 1...19. Thompson (1960 pp.120-121) already analyzed these expressions and showed their numerical equivalence. He makes reference to expressions including a word /tun/ which translate as "finalmente" or "ya no mas".

As coefficients of Long Counts (positional notation systems), there are several glyphs surveyed of course already by Thompson (1960). One functions also as a syllabary sign /mi/, and since the Yucatec word /mina'an/ means 'there is none', we can be relatively confident that this glyph refers to zero as absence, not completion. (See Thompson 1960 Fig.25 nos.55,56 and Fig.30 nos.1,2. He however on p.138, not knowing phonetic readings, argued that it must mean "completion", because it occurs in the half-period glyph which we also now know contains an /m/ -- TAHNLAM)

Another is the head form with hand covering lower jaw. This does not occur with month signs, so is ambiguous between completion and zero senses. (See Thompson 1960 Fig.25 nos.37-45.)

There is also a shell, a common glyph in place-notation numbers in the Dresden Codex (Thompson 1960 Fig.25 no.59).

And another form may be relevant here (Thompson 1960 Fig.25 nos.57,58). [check evidence here]

John Justeson's paper on a Place Notation before Zero follows here:

The topic of the concept of zero and its origin, in Mesoamerica and elsewhere, has been raised in a series of recent posts by Sam Edgerton, Paul Troemner, and Jorge Perez de Lara. They discuss when the long count was invented, but take for granted that the presence of the long count presupposes the prior invention of zero.

In a paper now in press, expanding on a 1981 talk, I argue for the opposite conclusion. Both on the Mesoamerican evidence itself and on the comparative evidence from the Old World, the concept of a numerical zero arises as a by-product of prior positional notation systems. The earliest positional notations do not use it;

rather, they use abacus-like formats in which no mark at all is used in the place of zero. Later, explicit marks are invented.

In Mesoamerica, the earliest positional notation is the long count date 7.16.2.3.13, on Chiapa de Corzo "Stela" 2 (actually a small panel). In the Mayan area in the Classic period, year-ending and katun-ending dates replete with zero digits are the most common, but they do not occur in the Late Preclassic nor in the epi-Olmec long counts, and in fact no zero digit is recorded in either group, although there are not so many such long counts that this could not be a chance effect (probability is between 5 and 10%).

Abaj Takalik Stela 5 is especially important in clarifying the origin of the Mesoamerican zero. This is a Mayan monument, and its text is in a Mayan language, though just which one cannot yet be determined. It presents two long count dates in strict positional notation, followed by the corresponding day in the ritual calendar -- but each long count contains only four digits each. The two dates are given as 8.3.2.10 then 5 (day sign) and 8.4.5.17 then 11 (day sign). The coefficients of the day signs abut directly against the day sign. The first can be restored as 8.3.2.[0].10 5 Dog, the second as 8.4.5.[0].17 11 Earthquake. Only blank space corresponds to the zero positions, which are aligned; and no zero sign had been present and lost, there is simply nothing there in the "uinals" position. Arguably, there is just a bit more space between the year and day positions than between any other pair of digits in the long count, but if so the effect is subtle. The two dates are laid out in parallel, with corresponding digits presented side by side.

For further details concerning the origins of a zero symbol and how empty space was used in the positional notation system we can only speculate, since there are so few early dates in positional notation and no other instance that would have called for a zero digit. But, at a minimum, the Abaj Takalik text shows that the positional notation of the long count does not presuppose the use of an explicit zero symbol. But the most straightforward interpretation is that no zero symbol existed in the earliest uses of positional notation. While there could be regional variability in its adoption, the use of an explicit mark for zero most likely postdates 125 AD, the later of the two dates on Stela 5. Since an explicit zero is attested at 8.14.0.0.0 (317 AD), it appears most likely that a sign for zero came into use among Mayans sometime in the second or third century AD. As yet we have no idea whether epi-Olmecs ever had a zero symbol, and, if they did, if they had it as early as Mayans did.

I don't feel that any importance attaches to the question of who first "invented" a zero symbol -- and I think that "discovered" is a more apt term. But since the issue has been raised, it appears that the Babylonians used a zero symbol a few centuries before southern Mesoamericans, and that it was first used in India a two or three centuries after it was in use in Mesoamerica (it was from India that it passed to the Arabs, and from them to Europe).

Regardless of the timing of the origin of zero, the important point for an understanding of this mathematical innovation is that it is the use of a symbol for zero that presupposes the prior development of a system of positional notation, not the reverse. All systems of positional notation in which zeros are found had previously existed for some time without it, and the quipu is an example of a system in which purely positional notation was used with no explicit symbol for zero.

- John Justeson